

Controllable single-photon frequency converter via a one-dimensional waveguide

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We propose a single-photon frequency converter via a one-dimensional waveguide coupled to a V -type atom. The on-demand classical field allows the atom to absorb a photon with a given frequency, then emit a photon with a carried frequency different from the absorbed one. The absorption and re-emission process is formulated as a two-channel scattering process. We study the single-photon frequency conversion mechanism in two kinds of realistic physical system: coupled resonator waveguide with cosine dispersion relation and an optical waveguide with linear dispersion relation respectively. We find that the driving field prefers weak in coupled resonator waveguide but arbitrarily strong in optical waveguide to achieve an optical transfer efficiency.

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I. INTRODUCTION

Photon frequency conversion [1–3] refers to transducing the input photons with a given frequency into the output photons with a different frequency while preserving the non-classical quantum properties. Experimentally, the photon frequency conversion has been achieved in the nonlinear medium by frequency mixing technologies [4–8]. In particular, Especially, the single-photon frequency conversion [9] has many applications in quantum information and quantum communication process, and the effective single-photon frequency conversion scheme [10] has been proposed in a waveguide channel with the assistance of the Sagnac interferometer which couples to a multi-level emitter [11–13].

Recently, the coherent control of the single photon has been studied in the one-dimensional (1D) waveguide with linear [14] and nonlinear [15–17] dispersion relations, where the two-level or three-level system acts as a quantum switch. When the three-level system is applied as a scatter [18–20], the photons can be transferred from one channel to another, and the carried frequency in different channels can be same or different, depending on whether the atom experiences the internal state transition in the scattering process or not. In present study, we aim to study a single-photon frequency converter mechanism in a single channel which is realized with waveguide.

To this end, we propose a scheme via a single waveguide with a located V -type atom, where the photon in the waveguide couples to one arm transition, and is scattered by the atom. To convert the frequency of the incident photon, we drive the other arm transition of the atom via a classical field. The subsystem consisting of the classical driving and the related two levels coupling to it, forms a pair of non-generated dressed ground states,

which supports two channels (but still in the same waveguide) for the incident photons. The energy conservation implies that the frequency of the incident photons will be converted when it is transferred from one channel to the other. We would remark that the V -type three-level system has been also used in Ref. [18], where the carried frequencies of photons in two channels are the same because only a single atom ground state is involved. Compared to the previous schemes [18], our scheme has the following two advantages: (i) We would apply only one single waveguide, which can be realized more easily with the current experimental technologies. (ii) The frequency difference between the two channels can be controlled only by adjusting the frequency and strength of the driving field, instead of the atomic energy level configuration.

In this paper, we firstly give a general description for the single-photon transmission and frequency conversion mechanism in one dimensional waveguide based on Lippman-Schwinger equation [21]. Then we consider two explicit models: the coupled resonator waveguide (CRW) [15–19] and one-dimensional optical waveguide [22–24]. For the case of CRW, the single-excitation spectrum of each of the two channels has the structure of one energy band and two discrete bound states with one of them being above the energy band and the other below it. On the contrary, the related spectrum structures of the two channels in the case of optical waveguide have only the lower energy bounds but no upper ones and bound states. In both of these models, the frequency converter only works when the incident photon frequency is inside the overlap regime of the spectra of the two channels. We find that the classical driving field prefers to be weak in the CRW but arbitrarily strong in optical waveguide to achieve an optimal transfer efficiency.

The rest of the paper is organized as follows. In Sec. II, we present the Hamiltonian in a general waveguide-atom coupled system, and give the scattering amplitudes based on Lippman-Schwinger equation. With these results, we discuss the single-photon scattering and frequency conversion in 1D CRW with cosine dispersion relation and in optical waveguide with linear dispersion relation in

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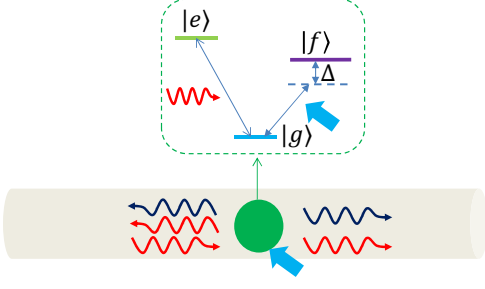


FIG. 1. (Color online) The schematic diagram of the model. A V-type atom is located in a 1D waveguide, serving as a frequency converter for the single photon injected from the left side of the waveguide.

Sec. III and Sec. IV, respectively. In Sec. V, we draw the conclusions.

II. FORMALISM OF THE FREQUENCY CONVERTER

A. The model and Hamiltonian

As shown in Fig. 1, the system we consider contains a 1D waveguide and a V-type atom. The V-type atom is characterized by a ground state $|g\rangle$, an intermediate state $|f\rangle$ and an excited state $|e\rangle$, whose energies are denoted as ω_g , ω_f and ω_e respectively. The energy of state $|g\rangle$ is set to zero as reference. The k th electromagnetic mode in the waveguide couples to the transition $|g\rangle \leftrightarrow |e\rangle$ with the strength J_k , while the classical driving field with frequency ν drives the transition $|g\rangle \leftrightarrow |f\rangle$ with the Rabi frequency η . In the rotating frame with respective $H'_0 = \nu|f\rangle\langle f|$, the Hamiltonian (with $\hbar = 1$) of the system is written as

$$H = \sum_k \omega_k a_k^\dagger a_k + \omega_e |e\rangle\langle e| + \eta(|g\rangle\langle f| + |f\rangle\langle g|) + \Delta|f\rangle\langle f| + \sum_k (J_k a_k |e\rangle\langle g| + J_k^* a_k^\dagger |g\rangle\langle e|), \quad (1)$$

where a_k is the annihilation operators for the k th mode of the electromagnetic field with frequency ω_k in the waveguide. The dispersion relation between the frequency ω_k and the wave vector k depends on the realistic physical system. $\Delta \equiv \omega_f - \nu$ is the detuning between the atomic transition $|g\rangle \leftrightarrow |f\rangle$ and the classical field.

The coupling between the classical field and the transition $|g\rangle \leftrightarrow |f\rangle$ forms two dressed states

$$|\phi_+\rangle = \sin \frac{\theta}{2} |g\rangle + \cos \frac{\theta}{2} |f\rangle, \quad (2a)$$

$$|\phi_-\rangle = -\cos \frac{\theta}{2} |g\rangle + \sin \frac{\theta}{2} |f\rangle \quad (2b)$$

with the corresponding eigenenergies

$$\nu_\pm = \frac{\Delta \pm \sqrt{\Delta^2 + 4\eta^2}}{2}, \quad (3)$$

where $\tan \theta = 2\eta/\Delta$. In the dressed-state representation, the Hamiltonian can be separated as $H = H_0 + V$: the free Hamiltonian

$$H_0 = \sum_k \omega_k a_k^\dagger a_k + \omega_e |e\rangle\langle e| + \sum_{i=\pm} \nu_i |\phi_i\rangle\langle \phi_i|, \quad (4)$$

for the waveguide and the atom, and the atom-waveguide interaction

$$V = \sum_k [J_k a_k (\sin \frac{\theta}{2} |e\rangle\langle \phi_+| - \cos \frac{\theta}{2} |e\rangle\langle \phi_-|) + h.c.] \quad (5)$$

B. The Lippman-Schwinger equation and scattering amplitudes

For a photon propagating in the 1D waveguide, when it is far away from the atom, the system is in free particle states governed by Hamiltonian H_0 . A photon coming toward the atom will first disappear, and appear later, i.e., the propagating photon is scattered by the atom. Taking the state $|k, \phi_n\rangle \equiv a_k^\dagger |0\rangle \otimes |\phi_n\rangle$ ($|0\rangle$ represents the photonic vacuum state in the waveguide and $n = \pm$) as the input state, which represents the atom in the internal state and the photon in the k th mode, the stationary state $|\psi_{kn}\rangle$ can be given by the Lippman-Schwinger equation [21, 25, 26]

$$|\psi_{kn}\rangle = |k, n\rangle + \frac{1}{E - H_0 + i0^+} V |\psi_{kn}\rangle. \quad (6)$$

Since the excitation number $N = \sum_k a_k^\dagger a_k + |e\rangle\langle e|$ is conserved in this system, the eigenstate with one excitation can be written as

$$|\psi_{kn}\rangle = \sum_p (\alpha_p |p, \phi_-\rangle + \beta_p |p, \phi_+\rangle) + u_{kn} |0, e\rangle. \quad (7)$$

Here, u_{kn} is the probability amplitude of the atom in the excited state, α_p (β_p) is the amplitude for finding one output photon with wave vector p in the waveguide and the atom in the state $|\phi_-\rangle$ ($|\phi_+\rangle$).

Combining Eqs. (7) and (6), and removing the photonic amplitudes, one can get the amplitude for the atom in the excited state

$$u_{kn} = \frac{(\sin \frac{\theta}{2} \delta_{n,+} - \cos \frac{\theta}{2} \delta_{n,-}) J_k}{\omega_{kn} - \omega_e - \sin^2 \frac{\theta}{2} A_+ - \cos^2 \frac{\theta}{2} A_- + i0^+} \quad (8)$$

with

$$A_\pm = \sum_p \frac{|J_p|^2}{\omega_{kn} - \omega_{p\pm} + i0^+}, \quad (9)$$

where we have defined $\omega_{kn} := \omega_k + \nu_n$.

In the following, we denote the state $|k', \phi_l\rangle = a_{k'}^\dagger |0\rangle \otimes |\phi_l\rangle$ ($l = \pm$) as the output state, i.e., a photon with wave vector k' is excited in the waveguide and the atom is in the internal state $|\phi_l\rangle$. Since the atom is in one of the dressed states in the presence of a photon of frequency ω_k before the scattering process, during the process it will absorb the photon with frequency ω_k and pass into the excited state $|e\rangle$, then emit the photon with frequency $\omega_{k'}$ and pass into any of the dressed states. The conservation of the total energy in the incoming and outgoing states offers a possibility for the outgoing photon carrying a frequency different from the absorbed one. After the scattering process, if the atom is found in the same state as the input state, i.e., $|\phi_l\rangle = |\phi_n\rangle$, we will have $\omega_{k'} = \omega_k$. However, if $|\phi_l\rangle \neq |\phi_n\rangle$ the frequency of photon will be changed ($\omega_{k'} \neq \omega_k$) since the eigenenergies of $|\phi_\pm\rangle$ are different. The evidence on the conservation of energy in the scattering process can be found in the following element of the S -matrix

$$S_{k', l \leftarrow k, n} = \delta_{l, n} \delta_{k, k'} - 2\pi i \delta(\omega_{k', l} - \omega_{k, n}) T_{k', l \leftarrow k; n}, \quad (10)$$

where the elements of the on-shell T matrix are obtained as [21]

$$\begin{aligned} T_{k', l \leftarrow k, n} &\equiv \langle k', \phi_l | V | \psi_{kn} \rangle \\ &= u_{kn} J_{k'}^* \left(\sin \frac{\theta}{2} \delta_{l, +} - \cos \frac{\theta}{2} \delta_{l, -} \right). \end{aligned} \quad (11)$$

In the above, the absorption and emission of single photons by the atom is formalized as a multi-channel scattering process. Here, there are two open channels with available states $|k, \phi_- \rangle$ and $|k, \phi_+ \rangle$, respectively. Hereafter, we will call them “negative channel” and “positive channel” according to the related atomic states $|\phi_- \rangle$ and $|\phi_+ \rangle$, respectively. In the following discussion, we restrict our consideration to the case that the single photon is incident from the negative channel with wave vector $k(> 0)$, then the element of the S -matrix in the negative channel is

$$S_{k', - \leftarrow k, -} = r_- \delta_{k', -k} + t_- \delta_{k', k} \quad (12)$$

where r_- (t_-) is the reflection (transmission) amplitude, and $t_- = r_- + 1$. The element of the S -matrix in the positive channel is

$$S_{k', + \leftarrow k, -} = t_+ [\delta_{k', q(k)} + \delta_{k', -q(k)}], \quad (13)$$

where the forward and backward transfer amplitudes are equal and denoted by t_+ . For the sake of simplicity, in what follows we will write the wave vector $q(k)(> 0)$ as q , whose dependence on the input wave vector k is given by the implicit relation

$$\omega_q = \omega_k - \nu_+ + \nu_-. \quad (14)$$

It shows from Eqs. (12,13) that, the wave vector of the scattering photon k' satisfies $k' = \pm k$ when the incident

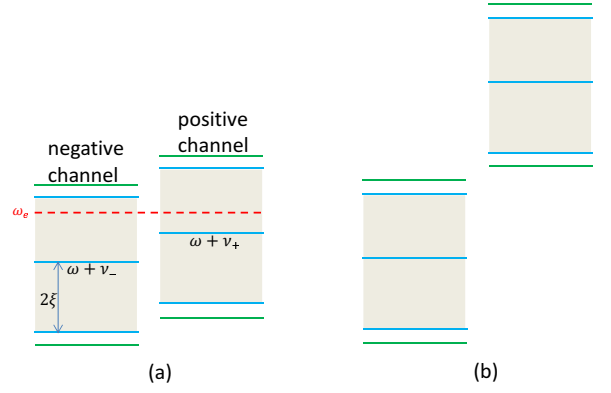


FIG. 2. (Color online) The energy band configurations for the two channels in the CRW. (a) The energy bands are partially overlapped. (b) The energy bands are completely separated. The thick lines represent the bound states in the corresponding channels and the red dashed line represents the energy of the atomic excited state $|e\rangle$.

photon is confined in the negative channel and $k' = \pm k$ when it is transferred to the positive channel.

It can be found from Eq. (10) that when the photonic flow is confined to the incident channel, the frequency of the emitted photon is equal to the absorbed one. However, when the incident photon is transferred to another channel, the frequency of the emitted photon will be lowered or raised by the amount $|\nu_+ - \nu_-|$. Consequently, the atom acts as a frequency converter for single photons propagating in the 1D waveguide.

III. SINGLE-PHOTON SCATTERING IN CRW

A 1D CRW is typically made of single-mode resonators that are coupled to each other through the evanescent tails of adjacent mode function of the cavity field. In experiments, the 1D CRW can be realized in photonic crystal, where the atom can be realized by a defect [27]. With the improvement of the fabrication techniques, a much stronger coupling between the atom and the waveguide mode has been realized in the superconductor transmission line, where the natural atom is also replaced by the superconductive qubit [28]. Assuming that all the resonators have the same frequency ω and the intercavity couplings ξ between any two nearest-neighbor cavities are the same, the 1D CRW is characterized by the dispersion relation

$$\omega_k = \omega - 2\xi \cos kl \quad (15)$$

where l is the lattice constance. In the rest of this section, the wave vector k is dimensionless by setting $l = 1$. For an atom located in the a th resonator, the atom-cavity coupling strength is $J_k = J e^{ika}$. Here, J is assumed to be real.

The energies for the free particle states in the negative (positive) channel form an energy band with the band-

width 4ξ , which is centralized at $\omega + \nu_-$ ($\omega + \nu_+$). The broken translation symmetry of the CRW allows bound states below and above the energy band of each channel [18]. Since $\nu_- \neq \nu_+$, by adjusting the strength and the frequency of the driving field, we could achieve two following band configurations as shown in Fig. 2: (a) partial overlap between the two bands; (b) no overlap between them.

For a propagating state excited in the negative channel, the reflection and transfer amplitudes are obtained from Eqs. (12,13) utilizing the residue theorem as

$$r_-^c = \frac{\cos^2 \frac{\theta}{2}}{2i\xi(\frac{\omega_k - \omega_e}{J^2} + \frac{i \sin^2 \frac{\theta}{2}}{2\xi \sin q}) \sin k - \cos^2 \frac{\theta}{2}} \quad (16a)$$

$$t_+^c = \frac{e^{i(|q|-k)a} \cos \frac{\theta}{2} \sin \frac{\theta}{2}}{2i\xi(\frac{\omega_k - \omega_e}{J^2} + \frac{i \cos^2 \frac{\theta}{2}}{2\xi \sin k}) \sin q - \sin^2 \frac{\theta}{2}} \quad (16b)$$

where the superscript “c” refers to the quantum channel made of “CRW” and the subscript $-$ ($+$) refers to the negative (positive) channel.

The cosine type dispersion in Eq. (15) characterizes the nonlinear relation between the frequency and the wave vector of the traveling photon in CRW. Therefore, for the incident photon with a fixed wave vector k , the group velocity is $v_g = 2\xi \sin k$. Meanwhile, the scattered photon in different channels will posse different group velocities. That is, $v_g = 2\xi \sin k$ in the negative channel and $v_g = 2\xi \sin q$ in the positive channel, where the wave vector q is defined in Eq. (14). Attentions should be paid here that the group velocities have the same unit with frequency.

In this sense, we define the scattering flows of the single photons as the square modulus of the scattering amplitudes multiplying the group velocities in the corresponding channels. For the sake of simplicity, we set the incident flow as unit and the reflection (transmission) flow in the negative channel is calculated as $J_c^r(J_c^t) = |r_-^c(t_+^c)|^2$ and the transfer flow to the positive channel is obtained as $J_c^{tr} = 2|t_+^c|^2 \sin q / \sin k$ or $J_c^{tr} = 0$ depending on whether the energy of the incident photon is inside or outside the energy band of the positive channel. It then follows from Eqs. (16) that the scattering flows are independent of the atom position.

Consider the case where the two energy bands are partially overlapped. Here, the incident frequency may be either inside or outside the continuous regime of the positive channel. In Fig. 3(a), we plot the reflection flow J_c^r , the transmission flow J_c^t , the summation $J_c^r + J_c^t$ in the negative channel, and the total flow $J_c^r + J_c^t + J_c^{tr}$ as a function of the incident frequency. It can be observed that the single photon is confined to the negative channel when the energy of the incident state is out of the overlap region of the two continuum bands. Consequently, the flow’s conservation is described by $J_c^r + J_c^t = 1$. When the incident energy is within the overlap region of the two continuum bands, the flow in the negative channel

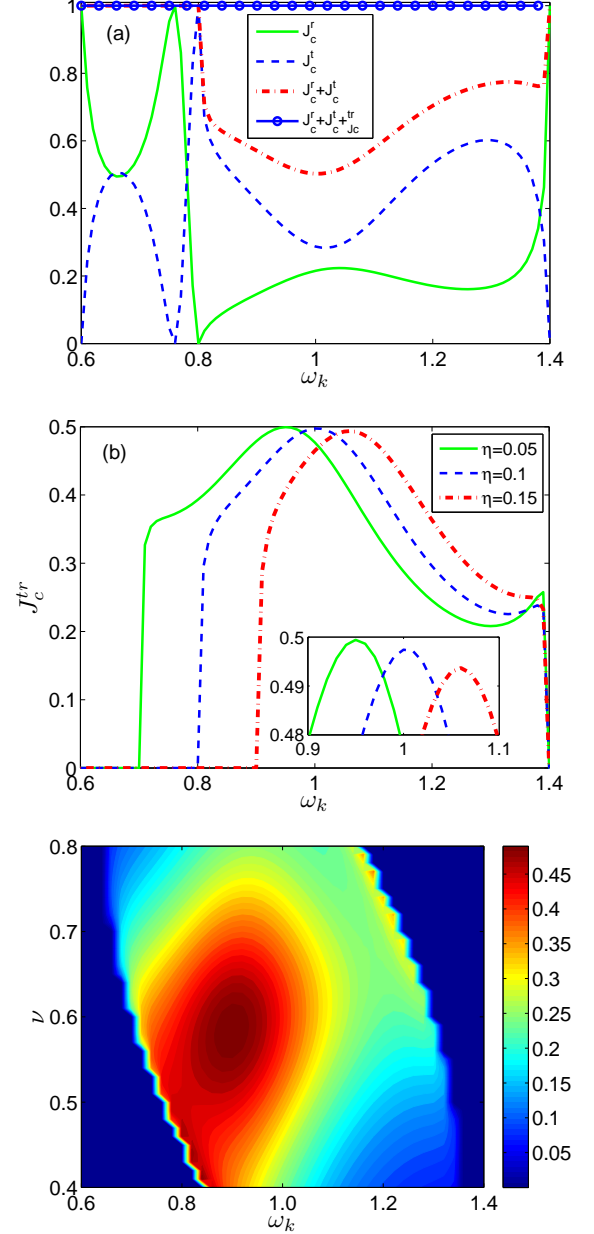


FIG. 3. (Color online) The scattering flows (in units of the incident flow) as a function of the incident frequency in CRW when the two energy bands are partially overlapped. (a) The reflection and transmission flows when $\Delta = 0$ and $\eta = 0.1$. (b) The transfer flows for different η under the resonance situation $\Delta = 0$. (c) The contour of the transfer flow versus ν and ω_k with $\omega_f = 0.6$ and $\eta = 0.1$. The other parameters are set as $\xi = 0.2$, $\omega_e = 0.9$ and $J = 0.3$ in units of $\omega = 1$.

satisfies $J_c^r + J_c^t < 1$, which means the incident photons can be transferred to the positive channel, and the flow conservation is changed as $J_c^r + J_c^t + J_c^{tr} = 1$. One can also find the complete reflection when the single photons are confined to the negative channel, which is the result of the interference between the open channel provided

by the negative channel and the close channel provided by the positive channel. This close channel is formed by a bound state, which is the consequence of the atom breaking the translation symmetry of the positive channel. The energy of the bound state in the positive channel can be obtained by the transcendental equation

$$\omega_{k-} - \omega_e = \frac{J^2 \sin^2(\theta/2)}{2i\xi \sin q}. \quad (17)$$

Actually, the bound state energy of the positive channel corresponds to the poles of the scattering matrix of the system studied in Ref. [15], i.e., a two-level system embedded in the positive channel.

In Fig. 3(b), we plot the transfer flow J_c^{tr} as a function of the frequency of the incident photon in the negative channel when the driven field is resonant with the atomic transition $|g\rangle \leftrightarrow |f\rangle$. It can be observed that when the frequency of the incident photon is resonant with the transition frequency between the atomic excited state $|e\rangle$ and the dressed state $|\phi_-\rangle$ ($\omega_k = \omega_e - \nu_-$), the transfer flow reaches its maximum. As the Rabi frequency η increases, the maximum of the transfer flow decreases monotonically. When the driving field is strong enough so that the two energy bands are completely separated (as shown in Fig. 2(b)), the transfer flow vanishes, i.e., the single photons can not travel in the positive channel. In Fig. 3(c), we have plotted the transfer flow versus the frequency ν of the driven field and the incident frequency ω_k by fixing the transition frequency $\omega_f = 0.6$ and the Rabi frequency $\eta = 0.1$ in units of $\omega = 1$. It can be found that the transfer flow reaches its maximum only when the frequency of the classical field is resonant with the atomic transition $|g\rangle \leftrightarrow |f\rangle$.

The above discussion shows that the overlap of the two bands is necessary for the atom to fulfill the function of a photonic frequency converter. Here, a weak driving field is preferred so that the frequency difference in the two bands can not be too large. However, the preference for weak driving field is not necessary when the CRW is replaced by a 1D waveguide with a linear dispersion relation, which will be discussed in the next section.

IV. SINGLE-PHOTON SCATTERING IN A ONE-DIMENSION OPTICAL WAVEGUIDE

In this section, we focus on the single-photon scattering in a 1D optical waveguide which naturally arises in a nanophotonic system. It exhibits a linear dispersion relation $\omega_k = v_g |k|$, where k is the wave vector and v_g is the group velocity of the light in the waveguide. We consider that the V-type atom is located at the position $x = a$ and the coupling strength between the atom and the waveguide is $J_k = J e^{ik a}$.

The energy of the free particle states in the negative (positive) channel have the lower energy bounds at ν_- and ν_+ respectively, but without the upper bounds, as shown in Fig. 4. For single photons with the wave vector

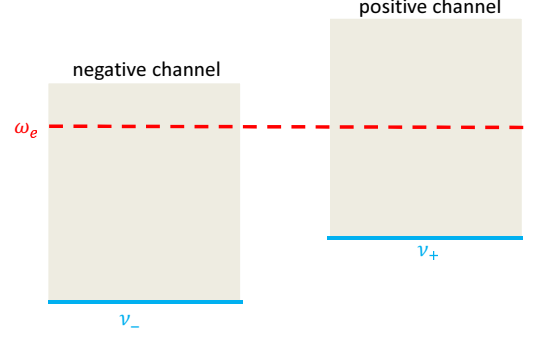


FIG. 4. (Color online) The energy level configurations for the two channels in the optical waveguide. The red dashed line represents the frequency of the atomic excited state.

k incident from the negative channel, the reflection and transfer amplitudes in the optical waveguide are calculated from Eqs. (12,13) utilizing the residue theorem as

$$r_-^o = \frac{J^2 \cos^2 \frac{\theta}{2}}{iv_g(\omega_{k-} - \omega_e)/L - J^2}, \quad (18a)$$

$$t_+^o = \frac{J^2 e^{i(|q|-k)a} \cos \frac{\theta}{2} \sin \frac{\theta}{2}}{iv_g(\omega_{k-} - \omega_e)/L - J^2}. \quad (18b)$$

Here, the superscript “o” implies the quantum channel made of “optical” waveguide and L is the length of the waveguide. The relationship between the wave vector q in positive channel and the wave vector k in the negative channel is obtained from Eq. (14) as

$$|q| = |k| - \frac{\nu_+ - \nu_-}{v_g}. \quad (19)$$

Similar to the last section, we can also define the scattering flows. Since the group velocities in both of the channels are the same, the reflection (transmission) flow in the negative channel is $J_o^r(J_o^t) = |r_-^o(t_-^o)|^2$, and the transfer flow in the positive channel is $J_o^{tr} = 2|t_+^o|^2$. It is same with the case in CRW that the scattering flows are independent of the atom position.

In Fig. 5, we plot the scattering flows as a function of the incident frequency with the assumption that the classical field resonantly drives the atomic transition. The reflection flow, transmission flow, the flow in the negative channel, and the total flow in the system is displayed in Fig. 5(a) and the transfer flow is depicted in Fig. 5(b). It can be observed from Fig. 5(a) [also in Fig. 5(b)] that when the incident frequency is below the difference between the lower bound of the positive and negative channels ($\omega_k < \nu_+ - \nu_- = 2\eta$), the photon is confined in the negative channel. Consequently, $J_o^{tr} = 0$ and the flow conservation equation is $J_o^r + J_o^t = 1$. However, the single photon is transferred to the positive channel once the incident frequency surpasses the energy difference of the lower bound of the two channels and the flow conservation equation becomes $J_o^r + J_o^t + J_o^{tr} = 1$. It is observed

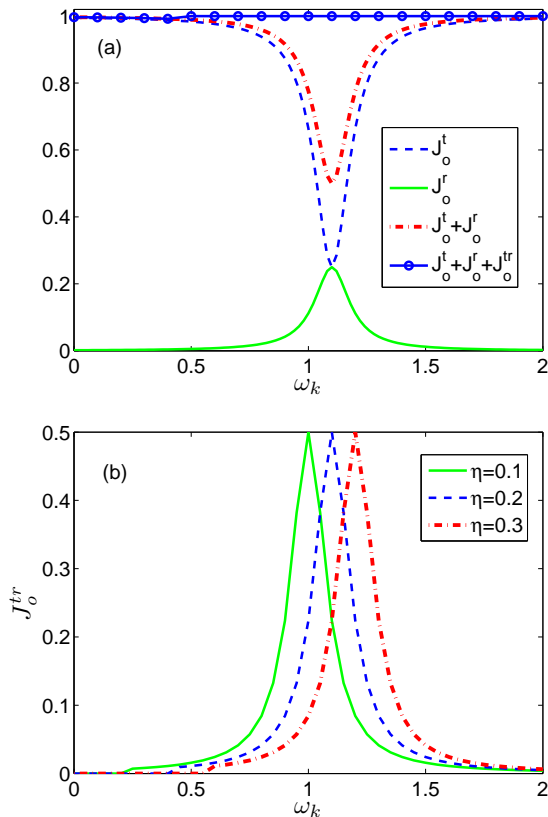


FIG. 5. (Color online) The scattering flows (in units of the incident flow) as a function of the incident frequency in the optical waveguide. The parameters are set as $\omega_e = 0.9$, $J = 0.3$, and $\Delta = 0$. All of the parameters are in units of $v_g/L = 1$. Under these parameters, the energy for the atom dressed ground states are $\nu_{\pm} = \pm\eta$. (a) The reflection and transmission flows when $\eta = 0.2$. (b) The transfer flows for different η .

from Fig. 5(b) that, the transfer flow achieves its maximum with magnitude 0.5 when the incident photon is resonant with the $|e\rangle \leftrightarrow |\phi_{-}\rangle$ transition, i.e., $\omega_k = \omega_e - \nu_{-}$. The maximum value of the transfer flow is independent of the strength of the driving light. When the frequency of the incident photon is far off resonance from the atomic transition, the transfer flow is close to zero, and the photon is nearly perfectly transmitted [as shown in Fig. 5(a)]. When the classical field is off-resonant with the atomic transition $|g\rangle \leftrightarrow |f\rangle$, the maximum of the transfer flow is lowered, i.e., always smaller than 0.5, which can be understood from Eq. (18b) and the relation $\tan\theta = 2\eta/\Delta$. For a resonant classical field, the detuning vanishes, and $\theta = \pi/2$, then the value of the numerator in Eq. (18b) achieves its maximum. For a off-resonant classical field

$\nu \neq \omega_f$, $\theta < \pi/2$. Consequently, the value of the numerator in Eq. (18b) is lowered.

The above discussion shows that the atom functions as a photonic frequency converter in 1D optical waveguide as long as the energy of the incident state lies inside the overlap region of the two channels. For single photons incident from the negative channel, it is possible to find that the carried frequency of the outgoing photon is red shifted. However, single photons incident from the positive channel can be also transferred to the negative channel, with the carried frequency of the outgoing photon being blue shifted instead of red shifted. We note that there is no total reflection below the lower bound of the positive channel, which means that there is no bound state for single photons in a waveguide with the linear dispersion relation. The requirement for converting the single-photon frequency is that the lower bound of the positive channel is below the transition frequency ω_e .

V. CONCLUSION

In this paper, we have studied the absorption and re-emission process of single photons by a V type atom embedded in a 1D waveguide. To show how the V-type atom behaves as a single-photon frequency converter by applying a classical field to drive the transition between the ground and intermediate state, we formulate the absorption and re-emission process as a two-channel scattering process. We analytically investigate the scattering flows in a 1D CRW with cosine dispersion and an optical waveguide with linear dispersion. It is found that converting the frequency of single photons in a 1D waveguide requires that (1) there is overlap between the continuums of the two channels; (2) the energy of the atom excited state is in the continuums of the two channels. As long as the requirements are satisfied, the strength of the classical field can be arbitrary large for the waveguide with linear dispersion relation to achieve the maximal probability of the outgoing photon with different frequency, however, a weak classical field is preferred in the 1D CRW.

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